PROTOTYPE EXAMINATION

Mathematics

Secondary Cycle Two
Secondary III

Competency 2
Uses mathematical reasoning

Name: _______________________________________

Group: ____________________________

School: __________________________________________

Time: 3 hours

Québec
Instructions

1. Fill in all the required information in the spaces provided in this booklet.

2. You may use a calculator (with or without a graphic display), a ruler, a set square, a compass, a protractor or additional graph paper.

3. You may refer to the memory aid you prepared on your own before the examination. This memory aid consists of one handwritten letter-sized sheet of paper (8½ × 11). Both sides of the sheet may be used. Any reproduction of this memory aid is forbidden.

Only the abovementioned materials may be used.

Note: Figures are not necessarily drawn to scale.
PART A

A B C D

1. 
2. 
3. 
4. 
5. 
6. 

PART B

7. There should be _______ men from the Hilltop borough in this sample.

8. There were _______ people in the group.

9. To the nearest cm², the lateral area of the smaller cone is _______ cm².

10. The mean finishing time of the runners in this marathon was _______ minutes.
PART C

This part of the examination consists of Questions 11 to 16.

For each question, you must show all your work to justify your answer.

Your work must be organized and clearly presented and cannot simply involve listing the calculator applications or programs used to obtain results or information.

The work you show on the even-numbered pages of this booklet will also be considered along with the work you show on the odd-numbered pages.

Each question in this part of the examination is worth 10 marks.

The following are the evaluation criteria for the competency required to answer the questions in this part of the examination.

Criteria 1  Formulation of a conjecture suited to the situation
Criteria 2  Correct use of appropriate mathematical concepts and processes
Criteria 3  Proper implementation of mathematical reasoning suited to the situation
Criteria 4  Proper organization of the steps in an appropriate procedure
Criteria 5  Correct justification of the steps in an appropriate procedure
11. **RAYMOND’S DRIVEWAY**

Raymond wants to have his driveway paved. He contacted two paving companies to find out how much it would cost. He was given the following information:

**COMPANY A**

The cost of paving a driveway is represented by function $f$ described below.

$$f(x) = 35x + 25$$

where $x$: area of the driveway, in $m^2$

$f(x)$: price, in $\$, charged by Company A to pave the driveway

**COMPANY B**

The paving cost includes a base price of $145 plus an amount proportional to the area of the driveway.

For example, Raymond’s neighbour paid Company B $1 696 to pave his 47 $m^2$ driveway.

Raymond has a rectangular driveway that is 6 m wide.

Both Company A and Company B would charge Raymond the same price to pave his driveway.

How long is Raymond’s driveway?
Raymond’s driveway is ______ m long.
12. **The Perimeter of Rectangle DEFG**

In the figure below, quadrilaterals ABCD and DEFG are rectangles.

The lengths of the sides of rectangles ABCD and DEFG can be represented by polynomials in which the coefficients of the terms and the constant terms are integers.

Note that:
- the length of segment DA is represented by a binomial
- the length of segment DC is represented by a monomial
- the area of rectangle ABCD is represented by the binomial $21x^2 + 15x$
- the perimeter of rectangle ABCD is represented by the binomial $20x + 10$
- $m_{DA} = m_{AE}$
- $m_{CG} = 2x + 6$

What binomial represents the perimeter of rectangle DEFG?
The binomial _______________ represents the perimeter of rectangle DEFG.
13. **It’s a Wrap!**

A piece of candy comes in a wrapper.

The piece of candy consists of a spherical piece of caramel covered with an even layer of chocolate.

The diameter of the spherical piece of caramel is 19.8 mm.

In this piece of candy, the volume of the piece of caramel is equal to the volume of the chocolate.

The candy wrapper is rectangular and 90 mm wide.

The area of the candy wrapper is 5 times the area of the piece of candy.

What is the length of the candy wrapper to the nearest tenth of a millimetre?
To the nearest tenth of a millimetre, the length of the candy wrapper is _______ mm.
14. **The Miniature Building Exhibit**

Gabriel will be renting space to exhibit miniature buildings.

Gabriel's profit is represented by function $f$ described below.

$$f(x) = ax - 1000 \quad \text{where } a \neq 0$$

- $f(x)$: Gabriel's profit, in dollars
- $x$: number of visitors to the exhibit
- $a$: admission fee, in dollars

Gabriel wants to know how many visitors he needs to break even.

Formulate a conjecture describing the change in the number of visitors Gabriel needs to break even after the admission fee is doubled.
Conjecture

After the admission fee is doubled, the number of visitors Gabriel needs to break even _______________________
______________________________________________________.
15. **SARAH’S TRIANGLES**

Sarah studied three triangles with the following characteristics:
- They are right triangles.
- In each triangle, the lengths of the two longest sides are two consecutive integers.

The following table indicates the lengths of the sides of the three triangles Sarah studied.

<table>
<thead>
<tr>
<th>Lengths of the Two Longest Sides</th>
<th>Length of the Shortest Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle 1: 13 m and 12 m</td>
<td>5 m</td>
</tr>
<tr>
<td>Triangle 2: 25 dm and 24 dm</td>
<td>7 dm</td>
</tr>
<tr>
<td>Triangle 3: 41 cm and 40 cm</td>
<td>9 cm</td>
</tr>
</tbody>
</table>

After calculating the length of the shortest side of each of the three triangles she studied, Sarah drew the following conclusion:

“If the lengths of the two longest sides of a right triangle are consecutive integers, then the length of the shortest side of the triangle will also be an integer.”

Is Sarah’s conclusion true or false? Explain why.
☐ Sarah’s conclusion is true.
☐ Sarah’s conclusion is false.

Explanation

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
16. **A SQUARE TARGET**

A square target is divided into three sections: one black, one white and one grey.

Each side of the target measures 28 cm.

This target is represented by square ABCD in the diagram below.

Quadrilaterals EFCG and HJCL are rectangles.

In addition:
\[ m \overline{LC} = 15 \text{ cm} \]
\[ m \overline{FJ} = m \overline{GL} = 3 \text{ cm} \]

A randomly thrown dart hit the target. The probability that the dart landed in the white section is less than the probability that it landed in the black section.

What are the possible lengths of segment JC?
The possible lengths of segment JC are ____________________
________________________________________ cm.